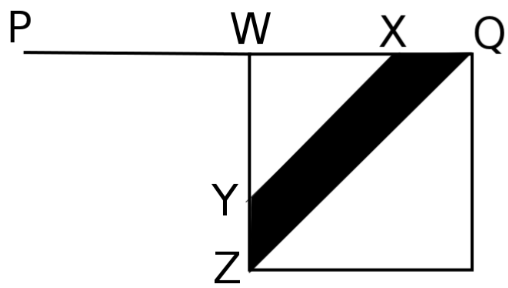
Stage 4 ★★
Mixed Selection 1 - Solutions

1. Quarters

The diagram shows the top-right-hand portion of the square.



The shaded trapezium is labelled QXYZ and W is the point at which ZY produced meets PQ. As QXYZ is an isosceles trapezium, ∠QZY = ∠ZQX = 45°.

Also, as YX is parallel to ZQ, ∠XYW = ∠WXY = 45°. So WYX and WZQ are both isosceles right-angled triangles. As ∠ZWQ = 90° and Z is at centre of square PQRS, we deduce that W is the midpoint of PQ. Hence WX = XQ = PQ/4. So the ratio of the side-lengths of similar triangles WYX and WZQ is 1:2 and hence the ratio of their areas 1:4.

Therefore, the area of trapezium QXYZ = $\frac{3}{4}$ x area of triangle ZWQ = $\frac{3}{32}$ x area PQRS since triangle ZWQ is one-eighth of PQRS. So the fraction of the square which is shaded is 4 x $\frac{3}{32}$ = $\frac{3}{8}$.

2. Angle to chord

Let O be the centre of the circle. Then $\angle POR = 90°$ as the angle subtended by an arc at the centre of a circle is twice the angle subtended by that arc at a point on the circumference of the circle.

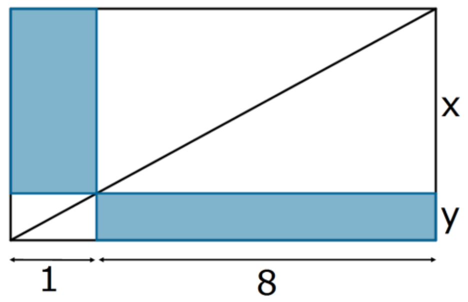
So triangle POR is an isosceles right-angled triangle with $PO = RO = 4\text{cm}$. Let the length of PR be x cm.

Then, by Pythagoras' Theorem, $x^2 = 4^2 + 4^2 = 2 \times 4^2$ and so $x = 4\sqrt{2}$

These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk)

3. Diagonal touch

Let x and y be the distances shown.



Then the shaded area is $8y + x$. But there are a number of similar triangles and from one pair
$\frac{x}{8} = \frac{y}{1}$ therefore $x = 8y$.

So,

$$\frac{shaded\ area}{total\ area} = \frac{8y + x}{9(x + y)} = \frac{8y + 8y}{9} \times 9y = \frac{16}{81}$$

4. Isosceles reduction

Triangles PRS and QPR are similar
because $\angle PSR = \angle QRP$ (since $PR = PS$)
and $\angle PRS = \angle QPR$ (since $QP = QR$).

Hence $\frac{SR}{RP} = \frac{RP}{PQ}$, that is $\frac{SR}{6} = \frac{6}{9}$, that is $SR = 4$.